On the definition of a general element and polarization

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Define $D_{yx} = y_1\frac{\partial}{\partial x_1} + \cdots + y_n\frac{\partial}{\partial x_n}$. This is called a polarization. In the classical invariant theory this is used to create a new invariant from a known invariant. In recent years this is used effectively in (1) and (2). In this talk I want to draw attention to the fact $D_{yx}, D_{xy}, [D_{yx}, D_{xy}]$ are an $\mathfrak{sl}_2$-triple acting on the homogeneous space of degree d of the graded algebra $K[y_1, \ldots, y_n][x_1 \ \ldots, x_n]$. It gives us new problems and new examples of Gorenstein algebras satisfying/failing SLP.

(1) P. Brändén and J. Huh, "Lorentzian polynomials," 2020.

(2) P. Marques Macias, M. McDaniel and A. Secelianu, "Higher Lorentzian polynomials, higher Hessians, and the Hodge Riemann property for graded oriented Artinian Gorenstein algebras in codimension two," 2024.