Maximal Hilbert functions of Artinian quotients of a product ring

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Given a field k and a graded k-algebra A, let $\mathbb{F}\Psi_A^{\mathbf{h}}$ and $\mathbb{H}\Psi_A^{\mathbf{h}}$ be the schemes parameterizing filtered quotients and graded quotients of A with Hilbert function $\mathbf{h}$. Let $\mathbb{F}\Psi_A^{\mathbf{h},\mathbf{t}}$ and $\mathbb{H}\Psi_A^{\mathbf{h},\mathbf{t}}$ be their subschemes of Artinian quotients of socle type $\mathbf{t}$.

In 1984, Iarrobino proved that, if k is infinite, if A is a polynomial ring, if $\mathbf{t}$ is permissible in a certain sense, and if $\mathbf{h} = \mathbf{h}^I$ where

$$\mathbf{h}^I(p) := \min\{a(p), sum_{q>0} t(q)a(q-p)\}$$

and $a(i) := dimA_i$, then $\mathbb{F}\Psi_A^{\mathbf{h},\mathbf{t}}$ is an affine space bundle over $\mathbb{H}\Psi_A^{\mathbf{h},\mathbf{t}}$, and $\mathbb{H}\Psi_A^{\mathbf{h},\mathbf{t}}$ is nonempty, irreducible and covered by open subschemes, each isomorphic to $\mathbb{A}^N$ with N explicit. For any A, there's a similar maximal $\mathbf{h}$, but it's not necessarily equal to $\mathbf{h}^I$.

In this talk, we analyze the case where $A := S \times_k T$ and $\mathbf{h} \neq \mathbf{h}^I$. When $S := k[x]$, a polynomial ring in one variable, we prove that $\mathbb{F}\Psi_A^{\mathbf{h},\mathbf{t}}$ and $\mathbb{H}\Psi_A^{\mathbf{h},\mathbf{t}}$ are close to be as nice as when $\mathbf{h} = \mathbf{h}^I$. In 2001, Cho and Iarrobino gave such examples with $T := k[y,z]/(z^5)$ in the graded case. The new work described here is joint work with Steve Kleiman.